FACULTY WORKING PAPER 91-0102

Gain/Loss Asymmetry in Risky Intertemporal Choice

Marjorie K. Shelley
Department of Accountancy

Bureau of Economic and Business Research
College of Commerce and Business Administration
University of Illinois at Urbana-Champaign

BEBR

FACULTY WORKING PAPER NO. 91-0102

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

January 1991

Gain/Loss Asymmetry in
Risky Intertemporal Choice

Marjorie K. Shelley

Department of Accountancy

This paper is taken from my dissertation completed at the University of Texas at Austin. I would like to thank my dissertation committee members, Urton L. Anderson, James S. Dyer, David V. Hinkley, Garry A. Marchant, and Lawrence A. Tomassini for their support and for their helpful comments throughout the project. I would also like to thank Don N. Kleinmuntz, George F. Loewenstein and Ira Solomon for helpful suggestions on earlier drafts of this paper. Finally, I would like to thank the colloquia participants at The University of Texas at Austin, the University of Illinois at Urbana-Champaign, The Ohio State University, the University of North Carolina, Duke University and the University of Kansas for insightful comments during the early stages of the work. The financial support of the Ernst & Whinney (Ernst & Young) Foundation is gratefully acknowledged.

Abstract

This paper investigates the claim that distant payoffs generate more risky choices than immediate payoffs. Decision makers will make more risky choices if loss discount rates are higher than gain discount rates or if the implicit risk of an option appears greater for loss than gain outcomes. These hypotheses were tested by comparing the loss and gain discount and implicit risk rates inferred from ratings of lotteries formed from a 4x4x4x2 (gain, loss, time, probability) experimental design. The ratings were used to estimate four alternative models of the lottery evaluation process. Implied loss discount rates were higher than gain rates, but the difference diminished substantially once the implicit risk rate had been extracted. Gain and loss implicit risk rates were also different and in a direction that implies greater risk tolerance for decisions with delayed consequences.

Conventional discounting models (e.g., the net present value model) imply constant discount rates across gain and loss, as well as over time. A constant rate is enforced when the conventional model is applied explicitly, but decision makers discount for delay and for implicit risk whether or not a formal model is invoked. The results of this study imply that decision makers' natural discounting processes do not conform to conventional theory. The departures from conventional discounting detected in this study can affect any intertemporal decision to which a formal model is not applied, the options selected to submit for formal evaluation, and the estimates made of potential outcomes and likelihoods for use in formal evaluations.

1. Introduction

"The riskiness of racetrack wagers declines as post time approaches. . . . Anxious public speakers are easier to sign up several months in advance than when the date of the speech is close at hand" (Jones & Johnson, 1973, pp. 613-614). Women who want to experience natural childbirth demand in advance that their doctors withhold pain medication to preserve the memory of the birth, but at the onset of labor, they renege and demand relief (Christensen-Szalanski, 1984; Schelling, 1984). Homeowners readily assume home equity loans with immutable five-year "balloon" payments, but regret their decision as the payment date draws near because, even though they expected the payment and knew its magnitude, they apparently failed to appreciate its eventual impact when they weighed it against the anticipated benefit.

These are anecdotal illustrations of what appears to be a discrepancy between the way decision makers weight gains relative to losses for immediate versus future outcomes. One explanation for the alleged asymmetry is a shift in the decision maker's reference point that occurs as the outcome approaches. For example, before the onset of labor, a woman's choice is between pain in the future, a negative but short-term outcome, and the experience and memory of natural childbirth, presumably a positive and long-term outcome. Once she is in labor, however, pain is the status quo - the reference point - and she must choose between relief, an immediate short-term positive outcome, and the experience of natural childbirth - two gains.

On the other hand, when risk is involved, the intertemporal difference in the cost/benefit balance could be due to decision makers' perceptions of risk; decision makers may exhibit an optimistic tendency to believe they can control the odds or the magnitude of potential future loss (March & Shapira, 1987). This perception would create a discrepancy between subjective discount rates for future negative and positive outcomes. Losses, pain, payments, etc., may lose their disutility faster into the future than gains of comparable magnitude lose their utility. A gain/loss discounting asymmetry has been noted in previous intertemporal choice research, but previous studies have not

usually addressed risk (Thaler, 1981; Benzion, Rapoport & Yagil, 1989; Loewenstein & Prelec, 1989b). The objective of this study is to estimate a descriptive model of an intertemporal lottery evaluation process and use the resulting parameter estimates to determine whether time preferences are influenced by asymmetric discounting when subject reference points are constant.

In what follows, Section 2 gives some background information concerning discounting theory and previous investigations of gain/loss asymmetry and implicit risk. Section 3 introduces four simple descriptive models of the subjective evaluation process for two-outcome risky decision options. The models' components are explained and specific hypotheses relating to predicted values of the estimated model parameters are discussed. Section 4 describes the experimental design and method of data analysis, Section 5 presents the experimental results, and Section 6 discusses the results, their implications, and directions for future research.

2. Background

2.1 Discounting

In general, an individual's "subjective rates of time preference are derived from his consumption utility function . . . They are independent of the market rates of interest and his borrowing and lending opportunities" (Henderson & Quandt, 1980, p.327). Since Fisher's (1930) formalization of discounted utility (DU), discount functions have usually been assumed to depend only on time distance, given a particular discount rate. But several departures from Fisher's model, including certain gain/loss asymmetries, appear consistently in recent literature (Thaler, 1981; Benzion et al., 1989; Loewenstein & Thaler, 1989; Loewenstein & Prelec, 1989a and 1989b; Shelley, 1990). Some of these departures were identified by early theorists as causes of inconsistent intertemporal choices and suboptimal planning (e.g., Strotz, 1955 and Thaler, 1981).

The departures from Fisher's DU theory noted in the literature tend to fall into the four categories identified by Loewenstein & Prelec (1989a & 1989b) as: (1) A common

difference effect - estimated discount rates fall as the length of delay increases; (2) an absolute magnitude effect - estimated discount rates fall as the absolute magnitude of the outcome increases; (3) delay/speed-up asymmetry, a framing effect involving a reference point shift - estimated discount rates vary according to whether consumption is delayed or expedited (e.g., for positive outcomes, the delay rate is greater than the speed-up rate); (4) gain/loss asymmetry - the balance between gain and loss outcomes is different for immediate versus future events.

Some inconsistencies in intertemporal decision making can be predicted by identifying particular departures from the conventional discounting model. For example: (1) If subjective discount rates vary inversely with time, decision makers will choose to delay the onset of adversity, to begin a savings program next month or a diet next week (Strotz, 1955; see also Thaler, 1981). If a decision maker's preferences among restaurants for dinner tonight depend on where he ate last night or where he will eat tomorrow, his discount rate depends on past or anticipated consumption and will not be constant over time as DU theory predicts. A decision maker who discounts loss faster than gain may prefer the lottery giving a fifty percent chance of gaining or losing $100 to the lottery giving a fifty percent chance of gaining or losing $200 if payoffs are immediate, but prefer the fifty percent chance of gaining or losing $200 if they are delayed for a year or two. Here, again, the balance between gain and loss is apparently different for future and immediate consequences. In this example, however, the two potential outcomes are not likely to be redefined as they were in the childbirth example. Losses remain losses and gains remain gains. Unfortunately, the decision maker who chooses the second lottery due to discounting loss faster than gain will ultimately want to renege.

Gain/loss asymmetry can account for a tendency to choose more risky options when outcomes are distant if the direction of the asymmetry implies faster discounting of loss than gain. But most evidence indicating a gain/loss asymmetry reported thus far has been generated in conjunction with the delay/speed-up asymmetry. The discrepancies

noted appear to have been caused by an interaction between outcome sign (i.e., whether the outcome is a gain or loss) and the direction of proposed changes in outcome timing (Thaler, 1981; Benzion et al., 1989; Shelley, 1990). The type of gain/loss asymmetry addressed in this study is strictly an outcome sign effect.

2.2 Implicit Risk

Implicit future uncertainty refers to risk that is distinct from the risk associated with lottery-type decision options. Risk attitudes for lotteries are reflected in the shape of the decision maker's expected subjective value/utility function. The uncertainty addressed by the implicit risk hypothesis exists in otherwise riskless, delayed decision options because the future is, by its nature, uncertain. Böhm-Bawerk (1923) posited that people charge a fee to compensate for the implicit risk in future events, a single proportional charge that, when combined with the time preference reduction, gives the overall discount on future goods.[1] He argued that this risk rate is independent of time discounting and, hence, has no "causal connection with the phenomenon of interest" as does the natural tendency to discount over time (Böhm-Bawerk, 1923, p. 247; see also Conard, 1963; Stevenson, 1986; Benzion et al., 1989).

The implicit risk hypothesis augments discounting theory by positing a premium that reduces the absolute magnitude of future gain and loss amounts according to the perceived uncertainty of future outcomes. If the perceived uncertainty is subjectively greater for loss than gain amounts, the discount will be asymmetric with respect to outcome sign and will cause the same effect as differential time discounting. Thus, a gain/loss asymmetry related to implicit risk could account for an apparent increase in risk tolerance for prospects with distant outcomes. Nothing in conventional theory predicts asymmetric treatment of the future uncertainty of gain and loss outcomes.

---

[1] An extension of the implicit risk hypothesis suggests that the premium is not a single one-time charge, but rather a charge that increases with time distance. However, although the implicit risk hypothesis was supported in both Benzion et al. (1989) and Stevenson (1986), the multiple-period extension has not been supported and is not addressed in this study (Benzion et al., 1989).

Yet despite the symmetric treatment of positive and negative outcomes in DU and expected utility theory, a large body of evidence exists that indicates decision makers rarely treat gains and losses symmetrically, whether choices involve delayed outcomes or not. For example, managers focus on the magnitude of possible negative outcomes to define risk (i.e., a risky option is one that contains the threat of a severe hazard), and they treat uncertainty about positive outcomes as an unimportant aspect of risk (March & Shapira, 1987).

What is necessary to distinguish the effects of differential time discounting, reference point shifts, implicit risk asymmetry, and inferred rate differences due to the shape of the value or utility function is: (1) an experimental approach that controls within-subject reference points, (2) a method of data analysis that can account for the effect of slope differences between the negative and positive portions of the subjective value function, and (3) a model that incorporates implicit risk.

2.3 Previous Empirical Studies

In a recent investigation of intertemporal choice, Loewenstein (1988) described three question frames that can be used to investigate intertemporal choice: a neutral frame, a delay frame, and a speed-up frame. Both the delay and speed-up frames induce reference point shifts that appear to increase implied subjective discount rates for positive outcomes relative to the rate produced using the neutral frame. Thaler (1981) found that delayed receipts produce higher implied rates than delayed payments. Benzion and colleagues (1989) found that both delayed receipts and expedited payments produce higher implied rates than expedited receipts or delayed payments. Although both Thaler and Benzion et al. conclude that receipts (positive outcomes) generate higher discount rates than payments, it is not possible, using their data, to separate the effect of outcome sign alone, from that of the reference point shift noted in Loewenstein (1988). It is clear from their evidence that subjective loss combinations (delayed gains and expedited losses) induce higher implied discount rates than

subjective gain combinations (delayed losses and expedited gains), but it is not clear that any discount rate asymmetries can be attributed to outcome sign alone (Shelley, 1990). This suggests that Loewenstein's concept of a neutral frame will be useful for isolating the effect of outcome sign alone on discount rates (Shelley, 1990). Both Thaler (1981) and Benzion et al. (1989) found that discount rates tend to vary inversely with time distance and absolute outcome magnitude.

In the early 1970s, several direct tests were made of the proposition that people choose more risky options when outcomes are delayed (Nisan, 1972; Jones & Johnson, 1973; Nisan & Minkowich, 1973). The results were mixed. Jones and Johnson (1973) and Nisan (1972) found fairly strong evidence in favor of the proposition. Nisan and Minkowich (1973) did not. The discrepancy in results appears to be associated with the interaction between the chosen dependent measure and the time delays included in the experiment. Subjective discount rates were not estimated in any of these early investigations even though the prediction in two relied on a theory that posited faster discounting of loss than gain outcomes (Miller, 1959).

3. Models and Hypotheses

3.1 Descriptive Models of Gain/Loss Asymmetry in Risky Intertemporal Choice Evaluation

Time Discounting: There are a number of candidates for a descriptive model of the intertemporal lottery evaluation process. If one or more of these models can explain (in a statistical sense) explicit evaluations of a set of N lotteries with M systematically varied payoff dates, then parameter estimates from fitting the model/s can be used to infer discounting patterns under uncertainty and, hence, to substantiate the existence of gain/loss asymmetry.

Let R_{ij} be a decision maker's response to a request for his evaluation of lottery i (i = 1, 2, . . . , N) at time j (j = 1, 2, . . . , M). It is assumed that for choosing among lotteries, knowing R_{ij} is equivalent to knowing $u(L_{ij})$, the subjective value of the lottery, L_{ij}, offering a p_k chance of gaining g_h and a $(1- p_k)$ chance of losing l_l, at outcome date j; so L_{ij} =

$(g_h, p_k, l_l; t = j)$ (with $h \times k \times l = N$). R_{ij} could be $u(L_{ij})$ or it could be the subjective equivalent value (SEV) of the lottery L_{ij}. Both expected utility theory and prospect theory recognize the existence of some number, labeled a subjective equivalent value herein, such that $u(L_{ij}) = u(SEV_{ij})$. In expected utility theory this number is called the certain equivalent value of the lottery. It can be expressed as a linear combination of the lottery objective values as long as the lottery outcomes are appropriately weighted. For a risk neutral individual, the SEV could be as simple a number as the expected value of the lottery. On the other hand, it may happen that, in arriving at a lottery's SEV, decision makers do not weight gain and loss the same or use probability decision weights that conform to objective probability values. Letting S(l) and S(g) represent a decision maker's evaluation of a lottery's gain and loss outcomes, $\pi(p)$ his probability decision weight, and $\delta_l(t)$ and $\delta_g(t)$ his loss and gain discount functions, respectively, one possible representation of his lottery evaluation is:

$$R_{ij} = u(L_{ij}) = SEV_{ij} = \delta_l(t)[1 - \pi(p)]S(l) + \delta_g(t)\pi(p)S(g) \qquad (1)$$

Subjective probabilities are allowed to vary from objectively-stated values in the estimation process to avoid distorting other parameter estimates. Implicit in this model is the assumption that the decision maker is risk neutral. In fact, Model (1) shows u(SEV) = SEV, so $u(\cdot)$ is the identity function. A linear subjective value function is an implicit assumption in much of the intertemporal choice literature in which discount rates are inferred; however, most of the studies that reflect this assumption have been conducted under certainty when it is more likely to hold because the concavity/convexity implied by risk aversion/risk seeking is not relevant. In Model (1), S(l) and S(g) are scaled (weighted) versions of the objective gain and loss outcome values, but they are linear functions of those values. If the linearity assumption fails and $u(\cdot)$ is (highly) nonlinear, the model's fit will be inadequate and estimates of its parameters could be seriously biased. When $u(\cdot)$ is allowed to be nonlinear (and otherwise different from the identity function), the model becomes

$$R_{ij} = u(L_{ij}) = u(SEV_{ij}) = u[\delta_l(t)[1 - \pi(p)]S(l) + \delta_g(t)\pi(p)S(g)]. \quad (2)$$

If $u(\cdot)$ is a prospect theory value function, it typically is predicted to be S-shaped, concave over gains, convex over losses, steeper for loss than gain outcomes, and gains and losses are defined as positive or negative departures from some relevant reference point. Expected utility theory suggests no particular shape. The subjective equivalent values (SEV_{ij}) remain linear functions of the lottery components, analogous to certain equivalents. If $u(\cdot)$ were known, the parameters comprising the SEV_{ij} values in Model (2) would be no more difficult to estimate, using nonlinear regression, than those in Model (1). Although $u(\cdot)$ is not known, it can be approximated at the same time the SEV parameters are estimated. Approximating the shape of $u(\cdot)$ enhances the model fitting process when the SEV model is the correct one. Hence, approximating $u(\cdot)$ can greatly reduce the possibility of biased parameter estimates for SEVs (Anderson, 1982).

Conventional discounting supports the model

$$R_{ij} = u[\delta(t)[1 - \pi(p)]S(l) + \delta(t)\pi(p)S(g)], \quad (3)$$

in which the gain and loss time parameters are constrained to be equal. Predicting a gain/loss asymmetry associated with time discounting implies that the gain and loss time parameters will not be equal; that is, $\delta_l(t) \neq \delta_g(t)$ in Model (2). If gain/loss asymmetry is an important aspect of a decision maker's evaluation, the fit of Model (3) should be significantly inferior to that of Model (2).

Implicit Risk: The implicit risk hypothesis asserts that the future is sufficiently uncertain that a one-time premium will be charged whenever an outcome is substantially delayed. Nothing in the theory suggests a gain/loss asymmetry associated with the premium. Neither the size, nor the occurrence, of future losses are predicted to be significantly less certain than the size, or the occurrence, of future gains. Therefore, the one-time discount rate for loss should be the same as the rate for gains. But according to March and Shapira (1987), decision makers are inclined to be much less concerned with the uncertainty associated with positive outcomes than with negative outcomes. If this

attitude extends to the uncertainty inherent in the future, decision makers may ignore implicit risk for gains, even though it is acknowledged for losses. Adding the implicit risk element to Model (2) produces the following model:

$$R_{ij} = u[\pi(p)\delta_g(t)\{\eta_g[\zeta(t)]S(g)\} + (1 - \pi(p))\delta_l(t)\{\eta_l[\zeta(t)]S(l)\}], \quad (4)$$

where $\eta_g[\zeta(t)]$ and $\eta_l[\zeta(t)]$ are the gain and loss implicit risk factors, respectively. The size of the implicit risk premium has tended to depend on the absolute magnitude of the outcome (Benzion et al., 1989). This implies a ratio function represented in Model (4) as $\eta_x(\zeta(t)) = e^{-d_x\zeta(t)}$. The risk rate is d_x (x = g, l); and $\zeta(t) = 0$ if $t = 0$, or $\zeta(t) = 1$ if $t > 0$. In the discrete case, $\eta_x(\zeta(t)) = (1 + d_x\zeta(t))^{-1}$.

Table 1 summarizes the model forms and distinguishing assumptions:

Table 1: Model Summary

Model	Symbolic Representation	Distinguishing Assumptions
(1) The Linear Model	$R_{ij} = u(L_{ij}) = SEV_{ij}$ $= \delta_l(t)[1 - \pi(p)]S(l) + \delta_g(t)\pi(p)S(g)$	The utility/value function is linear and discount rates are allowed to vary across gain and loss outcomes. The model assumes there is no implicit risk premium.
(2) The Full Model	$R_{ij} = u(L_{ij}) = u(SEV_{ij})$ $= u[\delta_l(t)[1 - \pi(p)]S(l) + \delta_g(t)\pi(p)S(g)]$	The utility/value function may be nonlinear and discount rates are allowed to vary across gain and loss outcomes. The model assumes there is no implicit risk premium.
(3) The Restricted Model	$R_{ij} = u(L_{ij}) = u(SEV_{ij})$ $u[\delta(t)[1 - \pi(p)]S(l) + \delta(t)\pi(p)S(g)]$	The utility/value function may be nonlinear; discount rates are <u>not</u> allowed to vary across gain and loss outcomes. The model assumes there is no implicit risk premium.
(4) The Implicit Risk Model	$R_{ij} = u(L_{ij}) = u(SEV_{ij})$ $u[\pi(p)\delta_g(t)\{\eta_g[\zeta(t)]S(g)\} + (1 - \pi(p))\delta_l(t)\{\eta_l[\zeta(t)]S(l)\}]$	The utility/value function may be nonlinear; discount rates are allowed to vary across gain and loss outcomes. The model allows for an implicit risk premium.

3.2 Hypotheses

Model (1) will be estimated and its fit compared with that of Model (2) to establish whether value functions are sufficiently nonlinear to distort discount rate estimates if nonlinearities are ignored. The distinctions among Models (2) through (4) were introduced to test the following hypotheses:

Gain/loss asymmetry for discount rates: Any important gain/loss asymmetry, associated with time discounting, will be detected by comparing the fit of Model (2) with that of Model (3), using an F statistic (Gallant, 1987). If there is gain/loss asymmetry, the fit of (3) will be inferior to that of (2). Conventional discounting theory predicts no difference. If there is a time-discounting gain/loss asymmetry for risky options, the prediction that risk tolerance increases with delay suggests that the direction of the asymmetry favors faster discounting of loss than gain outcomes ($\delta_l(t) < \delta_g(t)$, in Models (2) and (4), or letting $\delta_l(t) = e^{-st}$ and $\delta_g(t) = e^{-rt}$, $s > r$.

Implicit risk asymmetry: If the implicit risk hypothesis holds for delayed lottery payoffs, d_g and d_l will both be greater than zero. If there is no gain/loss asymmetry associated with implicit risk, $d_g = d_l$, but if decision makers ignore the future uncertainty associated with gain outcomes, d_g will be zero; hence, $d_g < d_l$, implying a gain/loss asymmetry for implicit risk. Model (4) separates the rate associated with implicit risk from that associated with time discounting. If significant differences were found between the time discount rates for gains and losses in Model (2), those differences will remain significant, after the implicit risk rate is extracted, if the length of payoff delay accounts for a substantial proportion of the predicted increase in risk tolerance associated with delayed consequences. Incidentally, estimated discount rates are expected to diminish with time as they have in previous studies (Thaler, 1981; Benzion et al., 1989; Shelley, 1990), so $r_t > r_{t+1}$ ($s_t > s_{t+1}$).

4. Experimental Method

4.1 Subjects

Thirty student subjects, from M.B.A. classes at the Graduate School of Business, The University of Texas at Austin, took part in the study. Subjects were paid $10 for each of two experimental sessions and were allowed to invest in a scaled version of one of the rated options as an incentive to rate honestly. The average subject payment was $25.33

for the two sessions; 26 of 30 subjects chose to participate in at least one incentive-scheme investment.

4.2 Design

Models (1) through (4) imply that, given objective values for payoffs, probabilities, and time, decision makers will transform the objective probability, time, and payoff values to subjective values and combine that information both multiplicatively and additively (across domains). A fully-crossed factorial design is nearly a requirement for testing the validity of such models (Anderson, 1982). For this study, a total of 128 prospect stimuli were formed by combining the levels of a 4 x 4 x 4 x 2 design. There were four gain ($1,000; $500; $100; $60), four loss (-$900, -$400, -$200, -$160), four time (immediate, six months, one year, and two years), and two probability (.6, and .4) levels. A total of 32 lotteries were formed at each of 4 outcome times. Gain and loss amounts were selected because they represent a range of values likely to be meaningful to the graduate student subjects. The experimental stimuli were presented in a neutral frame (i.e., no change in the expected outcome timing was proposed; see Loewenstein, 1988 and Shelley, 1990), and care was taken to insure a stable within-subject reference point on the scale. Because each rated option contained both the gain and loss aspects of interest, reference points were also stable across gain and loss outcomes.

4.3 Training and Task.

The first 30 to 45 minutes of the first experimental session were used to read and sign a participation contract, instruct each subject in the use of the rating scale, and demonstrate the incentive scheme. Once subjects were comfortable with their use of the rating scale, a spiral-bound practice booklet was used to demonstrate the experimental incentive scheme and to insure that subjects understood the task and its relation to the incentive scheme. Subjects were allowed to make changes in their use of the scale following the practice session.

Figure 1 is a sample stimulus from an experimental booklet. Both accounting gain and loss and cash flow information are provided. Subjects had previously identified the worst and best options in the set and written the relevant information about them on a reference rating scale. Because investment cost was the same across all options, it should have been cancelled in the editing phase of the subjective evaluation stage so that options were evaluated based on their outcome magnitudes, probability levels, and outcome times (Kahneman & Tversky, 1979).

Page 19

This investment option offers you a six in ten chance of gaining \$1,000 and a four in ten chance of losing \$900. Whether you lose or win, final payments will be made immediately. The cost of the investment is \$100.

Cash flow: The \$100 cost is paid immediately.
If the outcome is:
1. A gain - Your investment cost is returned and you receive an additional \$1,000 immediately.
2. A loss - Your investment cost is not returned and you pay an additional \$800 immediately.

RATING SCALE

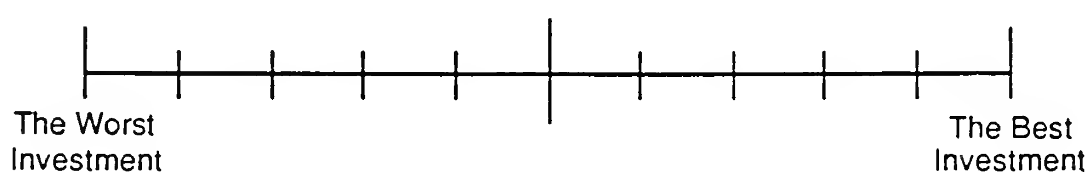

Figure 1
Sample Stimulus

In the proposed descriptive models subjects' responses, the evaluative ratings, R_{ij}, are assumed to result from (1) transforming (or scaling) the objective values of the independent variables to subjective values and (2) combining the subjective values in a particular way. Because subjects' direct magnitude estimates of the values $u(L_{ij})$ tend to be biased and do not give a reliable scale (Anderson, 1982), subjects were asked to rate

the relative attractiveness of the lotteries on a graphic rating scale rather than estimate the values $u(L_{ij})$. The observed ratings, R_{ij}, were assumed to be strategically equivalent to the values $u(L_{ij})$.

The experiment was run with one subject at a time. Subjects were asked to rate all 128 prospects in each of two separate sessions set one day apart. Subjects were allowed to make computations in their experimental booklets. Obtaining responses from two sessions mitigated random inconsistencies and possible order effects. It also enabled a measure of pure error for testing model fit (Draper & Smith, 1981). Prospect orderings were random within the experimental booklets and a different random ordering was used for each session. Each experimental session took one hour to complete, on average.

4.5 Analysis

Model (1) implies that lottery ratings are continuous, monotone linear functions of experimental objective values. If linearity is assumed in error, (1) ANOVA tests of the appropriateness of the SEV model will be misleading, (2) tests of the model's lack of fit will be significant, and (3) parameter estimates may be seriously biased. The parameter estimates from fitting Models (2) through (4) reflect each subject's actual value function because their value functions were approximated as parameters were estimated (using integrated, second-order, normalized B-splines; deBoor, 1978; Winsberg & Ramsay, 1981; Schumaker, 1981; Stevenson, 1986). All models were fit using Marquardt's compromise procedure as the nonlinear regression algorithm; outcome magnitudes were constrained to fall between 10 (for the \$1000 outcome) and -9 (for the -\$900 outcome); and the immediate time parameter was fixed at one (e.g., $\delta(t) = e^{-rt} = 1$ at $t = 0$). To test the time parameter predictions specified in the previous section, gain and loss time parameters were estimated for each subject and each model. Implied rates were computed using the continuous compounding formula, $\hat{\delta}_g(t) = e^{-\hat{r}t}$ ($\hat{\delta}_l(t) = e^{-\hat{s}t}$). Parameter estimates were compared across the linear and approximation models; the fit

for each model was tested for each subject, and the fit of the restricted Model (3) was compared with the fit of the full Model (2).

5. Results

5.1 Tests of Scale Use and Model Fit.

Scale Use: If the rating scale (see Figure 1) was used correctly, dependent measures can be interpreted as continuous, monotone increasing functions of the subjective equivalent values of the lotteries. To check that all 30 subjects used the scale in the same way, a factor analysis was conducted using subjects as variables and mean (across sessions) ratings as observations (128 observations on 30 variables). If all subjects used the rating scale in the same way, they should all load significantly on a single factor. All subjects did, in fact, load significantly on one factor. The factor loadings ranged from a low of 0.725 to a high of 0.976. The mean factor loading was 0.91. Subjects appear to have used the rating scale appropriately and the data of every subject who completed the experiment were used in the model fitting process.

Model Fit: A plot of observed ratings against expected values for the 32 immediate and 32 two-year lotteries is shown in Figure 2 for Subject 8. A line for each outcome time indicates where ratings would fall if they were linear functions of expected values. Subject 8's value function appears to be approximately S-shaped in this plot. Value functions for 24 of 30 subjects revealed striking nonlinearities on visual inspection.

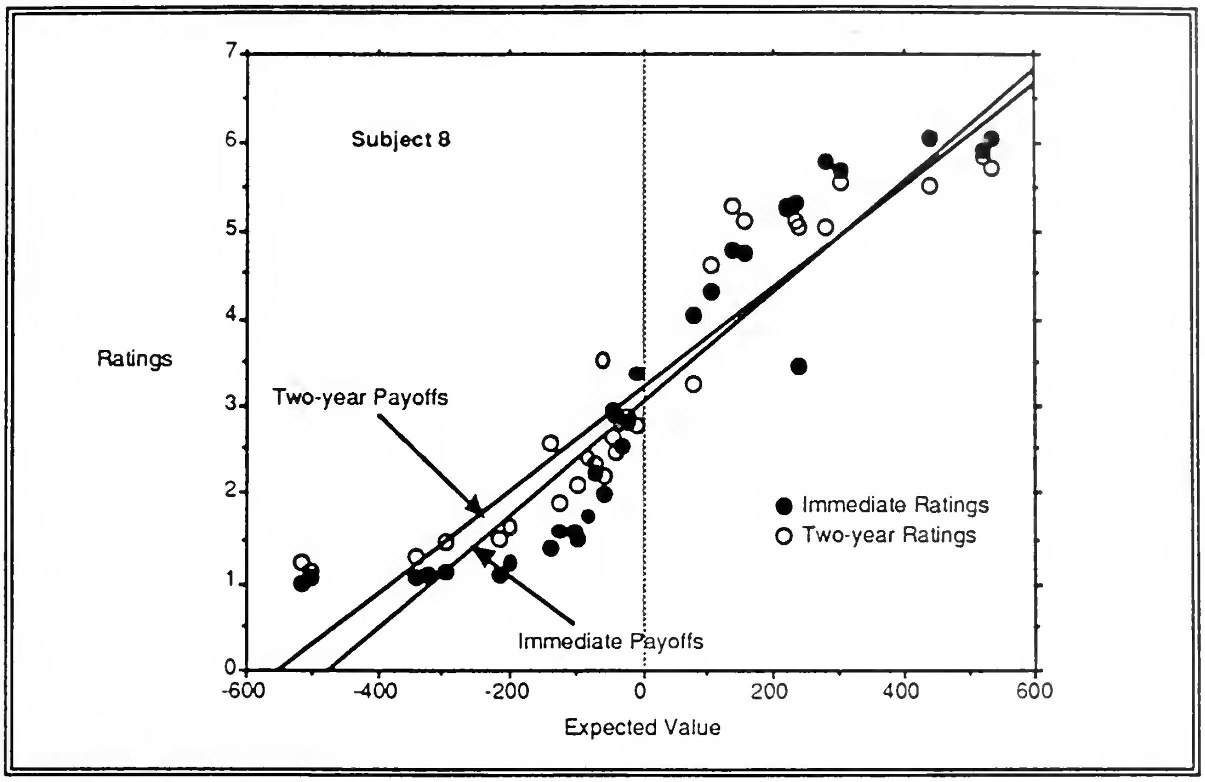

Figure 2
S-shaped Value Function

The results of an ANOVA (Table 2) indicate that, if a linear value function is assumed (Model (1)), the data do not support the proposed SEV model. Alternatively, if the SEV model is correct, the data do not support a linear value function. This alternative is consistent with the visual inspection of plots of raw ratings against expected values (e.g., Figure 2). All the proposed models imply that in arriving at an SEV value, (1) probability, time, and subjective gain and (2) probability, time and subjective loss combine multiplicatively, but (3) the loss and gain components are combined additively. If these assumptions hold, enough levels of the explanatory variables (factors) are effective, and u(·) is linear, a repeated measures ANOVA using the implicit values, $u(L_q)$, $q = N \times M$, as dependent measures would reveal significant two-factor interactions for every combination of two factors, except gain and loss because gain and loss are summed to arrive at $u(L_q)$. There should also be reliable three-factor interactions, except for combinations that include both gain and loss. The four-factor interaction would not be significant. However, even if the SEV model ($SEV_{ij} = \delta_l(t)[1 - \pi(p)]S(l) + \delta_g(t)\pi(p)S(g)$) is

Table 2
Univariate ANOVA Results
Dependent Measures = Rating Values (R)

Source	df	F	Prob > F
Prob x Time	3	4.49	0.0056
Prob x Gain	3	46.80	0.0001
Prob x Loss	3	8.59	0.0001
Time x Gain	9	2.86	0.0031
Time x Loss	9	3.06	0.0017
Gain x Loss	9	32.53	0.0001
Prob x Time x Gain	9	1.17	0.3133
Prob x Time x Loss	9	1.03	0.4126
Prob x Gain x Loss	9	3.37	0.0006
Time x Gain x Loss	27	2.57	0.0001
Prob x Time x Gain x Loss	27	1.99	0.0021

adequate and the factor levels are effective, u(·) could be nonlinear. If u(·) is highly nonlinear, interactions are less predictable. Some expected interactions may fail to reach significance and some interactions may appear across the additive components of the model.

As Table 2 indicates, the predicted two-way interactions are significant, but there is also a strong two-way interaction between gain and loss. The three-way prob x gain x loss and time x gain x loss interactions are also significant. No other three-factor interaction is significant. Because nonlinearities were obvious for many subjects, the invalid assumption implied by the ANOVA results above (Table 1) is taken to be inappropriately assuming a linear value function rather than inappropriately specifying the SEV model. Tests for lack of fit for Model (1) were significant for all but four subjects.

Because the evidence favors nonlinear value functions, each subject's value function was approximated in fitting Models (2) through (4). With the spline approximation, Models (2) and (4) produced an adequate fit for 27 of 30 subjects. Statistics for lack of fit and the percent of explainable variation explained are shown in the Appendix for Model (2). The approximated value functions fell generally into three categories: (1) linear (6 subjects), (2) S-shaped (10 subjects), and (3) convex (14

subjects). Figure 3 shows a convex function that resembles the loss portion of a prospect theory value function. Figures 4 and 5 show linear and S-shaped value functions, respectively.

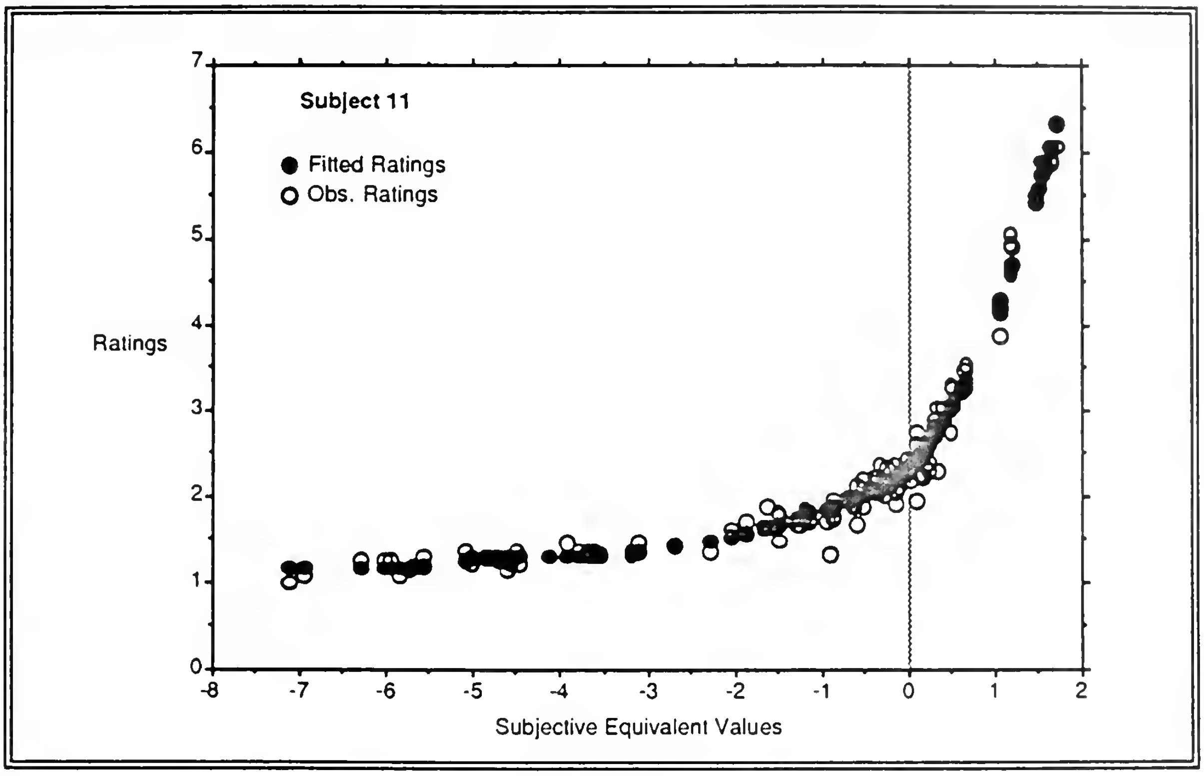

Figure 3
Approximated Convex Value Function

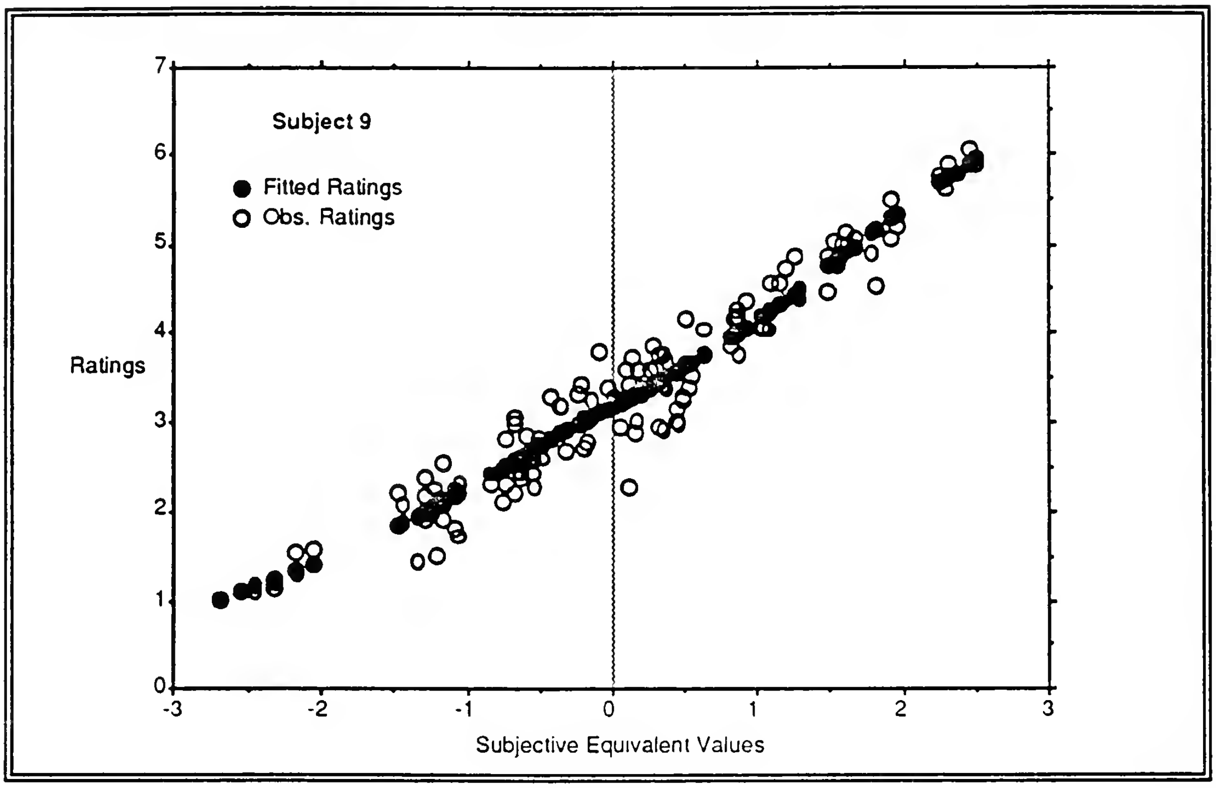

Figure 4
Linear Value Function

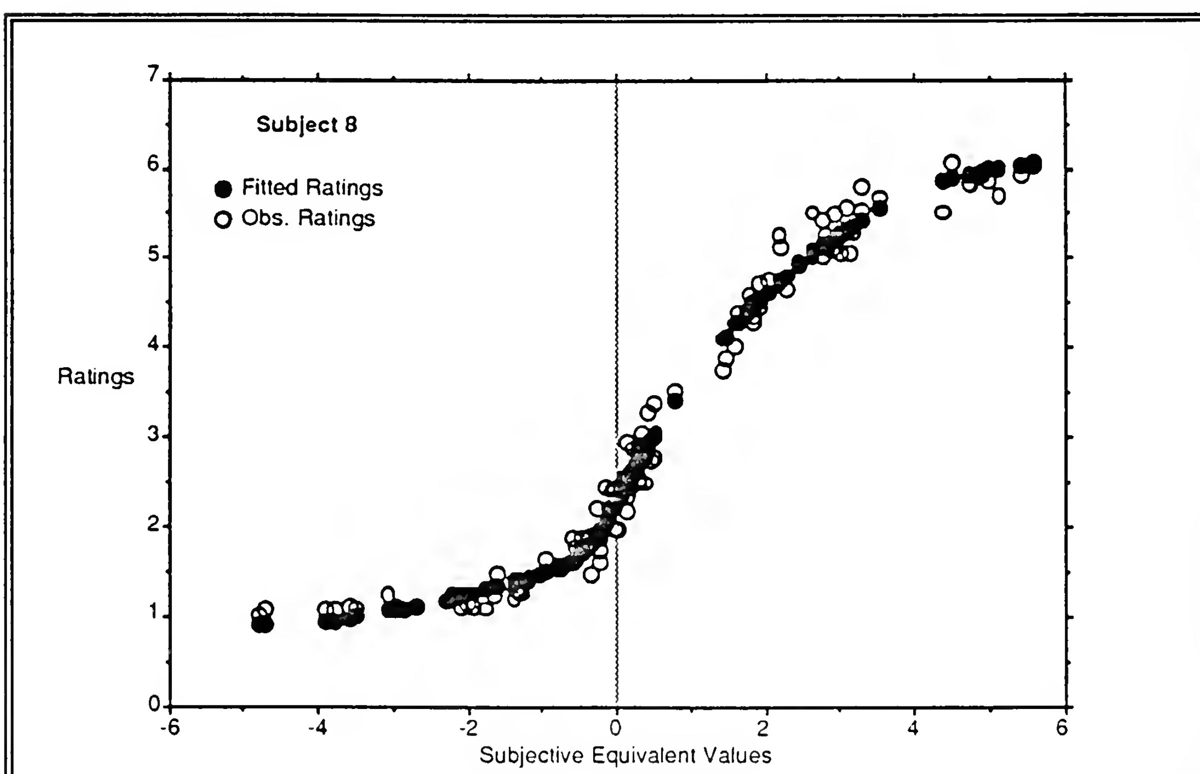

Figure 5
S-Shaped Value Function

5.2 Hypothesis Tests

Table 3 - Model (1): Linear Value Functions
Mean Discount Rates by
Outcome Sign and Delay Length

Domain: Time	**Est. Mean Discount Rate**	**Std. Dev.**	**Std. Error**	**Variance**
Gain:				
Six Months	0.117	0.072	0.013	0.005
One Year	0.060	0.036	0.001	0.007
Two Years	0.032	0.026	0.001	0.005
Loss:				
Six Months	0.277	0.241	0.044	0.058
One Year	0.207	0.391	0.071	0.153
Two Years	0.075	0.053	0.009	0.003

Mean discount rate estimates (across subjects), standard deviations, standard errors and variances for each outcome sign and delay combination are shown in Tables 3 and 4 for Models (1), (2), and (4). Rate estimate differences between Model (1) and Models (2) and (4) are greater for loss than gain payoffs, and the standard errors of the rate estimates from Model (1) are biased downward making the null hypotheses too easy to reject.

As shown in Table 4, mean gain discount rate estimates are about the same whether an implicit risk rate is extracted or not, but mean loss discount rate estimates are reduced substantially when the implicit risk rate is extracted. Loss rate estimates, compared across the implied-risk and no-implied-risk models ((4) and (2), respectively), are significantly different for all three delay lengths ($p = 0.0236$ for six months, $p = 0.0239$ for one year, and $p = 0.0321$ for two years), but gain rates are not significantly different across the two models.

Table 4 - Approximated Value Functions
Mean Discount Rates by
Outcome Domain and Delay Length

Domain: Time	**Est. Mean Discount Rate**	**Std. Dev.**	**Std. Error**	**Variance**
Gain:				
Six Months				
No Risk Rate Extracted (2)	0.109	0.169	0.026	0.020
Risk Rate Extracted (4)	0.108	0.144	0.027	0.021
One Year				
No Risk Rate Extracted (2)	0.060	0.078	0.014	0.006
Risk Rate Extracted (4)	0.057	0.048	0.009	0.002
Two Years				
No Risk Rate Extracted (2)	0.039	0.047	0.009	0.002
Risk Rate Extracted (4)	0.040	0.032	0.006	0.001
Loss:				
Six Months				
No Risk Rate Extracted (2)	0.223	0.313	0.057	0.098
Risk Rate Extracted (4)	0.149	0.203	0.037	0.041
One Year				
No Risk Rate Extracted (2)	0.114	0.168	0.031	0.028
Risk Rate Extracted (4)	0.074	0.097	0.018	0.009
Two Years				
No Risk Rate Extracted (2)	0.073	0.082	0.015	0.007
Risk Rate Extracted (4)	0.056	0.053	0.010	0.003

Tests for a difference in fit between the restricted and full models (Models (3) and (2), respectively) indicate that the restricted model's fit is significantly inferior ($p < .10$) to that of the full model for 12 of 30 subjects, so some gain/loss asymmetry is present in the discounting process. In fact, seven subjects discounted loss faster than gain in general, one subject discounted gain faster than loss, and four subjects discounted asymmetrically only in one period. For example, Subject 30 stated in her exit interview that she had a particular aversion to any loss that might occur approximately one year in the future, and, in fact, Subject 30's ratings implied an extremely high negative discount rate for losses delayed one-year.

The 30 gain and 30 loss discount rate estimates for each delay length allowed three paired comparison t-tests of the null hypothesis $r_t = s_t$ against the alternative $r_t < s_t$. Time parameter estimates from both Models (2) and (4) were used to test whether, on average, loss or gain discount rates are higher. The results are shown in Table 5. When no risk rate is extracted, gain and loss discount rates differ significantly for each delay length indicating a gain/loss asymmetry. The direction of the difference supports the prediction that choices will be more risky when outcomes are distant.

Table 5
Hypothesis Test Results:
Loss Discount Rates Are Higher Than Gain Rates

Test:	Mean Difference	df	Paired t-value	Prob > t One-tail
$r_t < s_t$				
Six Months				
No Risk Rate Extracted	-0.113	29	-2.820	0.0043
Risk Rate Extracted	-0.041	29	-1.649	0.0550
One Year				
No Risk Rate Extracted	-0.054	29	-1.990	0.0281
Risk Rate Extracted	-0.017	29	-0.989	0.1655
Two Years				
No Risk Rate Extracted	-0.034	29	-2.373	0.0123
Risk Rate Extracted	-0.016	29	-1.908	0.0332

With the implicit risk rate extracted, the gain and loss discount rates for the one-year delay are no longer significantly different. However, the pattern that remains tends to support a gain/loss asymmetry with losses discounted faster than gains. The discount rates shown in Table 4 indicate that the implicit risk hypothesis may hold for possible future losses because loss discount rates change significantly when the implicit risk rate is extracted; gain rates appear virtually unaffected. Tests of the prediction that both the gain and loss risk rates are greater than zero are consistent with this observation. Only the implicit risk rate for losses is significantly greater than zero ($p = 0.0152$). It appears that the uncertainty inherent in the future is acknowledged for loss outcomes, but not for

gain outcomes. It follows that the loss risk rate is significantly greater than the gain rate ($p = 0.0146$).

The independence assumption reflected in the conventional discounting model implies constant-rate discounting over time, but previous evidence (Thaler, 1981; Benzion et al., 1989; Shelley, 1990) suggests that discount rates tend to decline with delay length. This has been labeled the "common difference effect" (Loewenstein & Prelec, 1989a) and has been identified as one cause of dynamic intertemporal inconsistency (Strotz, 1955; Thaler, 1981). Some of the apparent decline may be caused by rate estimates that include both the one-time risk rate and the time preference rate. Table 6 shows test results for the null hypothesis that subjective discount rates do not vary with delay length.

Table 6
Hypothesis Test Results:
Discount Rates Diminish Over Time

Test:	Mean Difference	df	Paired t-value	Prob > t One-tail
$r_t - r_{t+1} > 0$				
t = Six Months				
No Risk Rate Extracted	0.049	29	2.046	0.0249
Risk Rate Extracted	0.051	29	2.105	0.0220
t = One Year				
No Risk Rate Extracted	0.022	29	2.290	0.0148
Risk Rate Extracted	0.017	29	1.966	0.0295
$s_t - s_{t+1} > 0$				
t = Six Months				
No Risk Rate Extracted	0.109	29	2.268	0.0155
Risk Rate Extracted	0.076	29	1.673	0.0526
t = One Year				
No Risk Rate Extracted	0.041	29	2.017	0.0265
Risk Rate Extracted	0.018	29	1.308	0.1006

As in previous investigations, the evidence suggests that subjective discount rates vary inversely with the length of delay; however, the decline in loss rates is not significant once the implicit risk rate has been extracted. Figures 6 and 7 are graphic illustrations of the decline in rates over time for the no-implicit-risk and implicit-risk fits (Models (2) and (4)), respectively. In Figure 6 gain and loss discount rates appear to be converging, but Figure 7 shows that once the implicit risk rate has been extracted, gain and loss rates appear on visual inspection to decline at about the same pace.

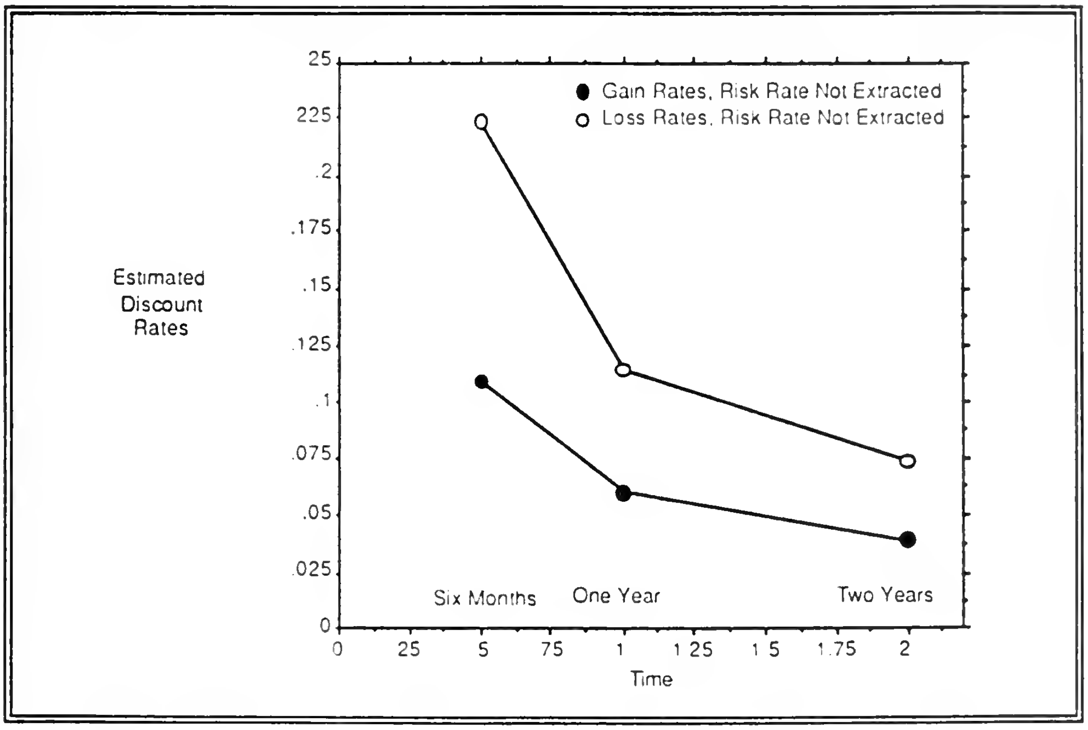

Figure 6
Gain and Loss Discount Rate Decline
Over Time: No Implicit Risk Rate Extracted

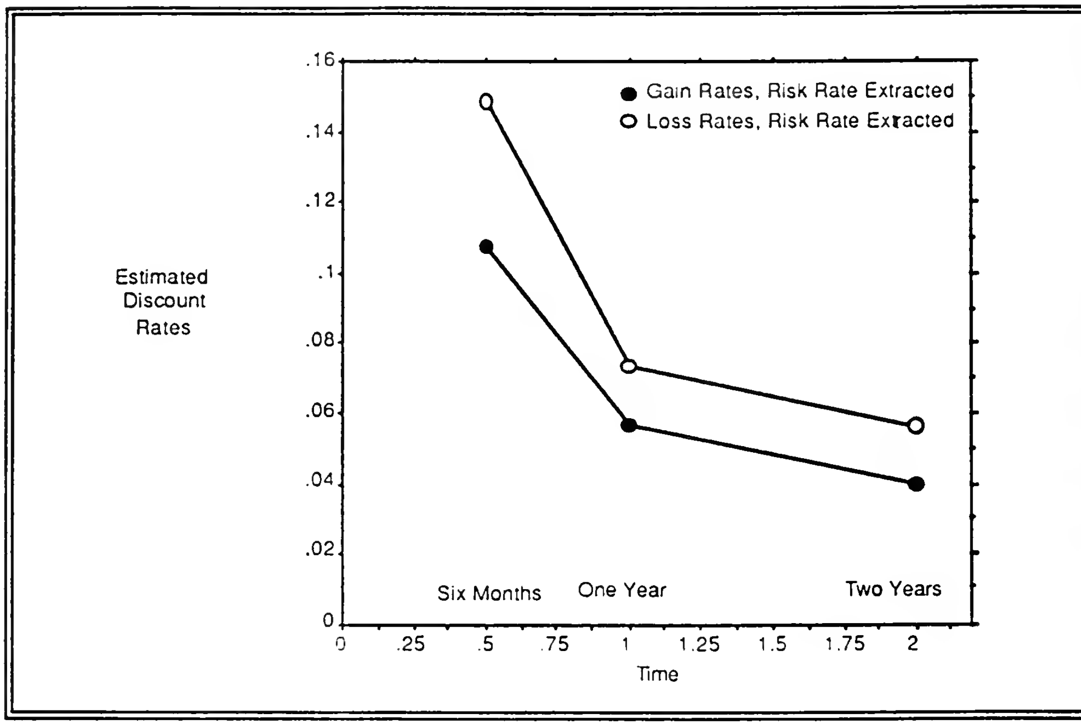

Figure 7
Gain and Loss Discount Rate Decline
Over Time: Implicit Risk Model

Hypothesis test results support the presence of gain/loss asymmetries for both time discounting and implicit risk for the lottery evaluation task. Parameter estimates from Model (4) imply that losses are discounted more than gains for both time and implicit risk, so either, or both, discounts will explain an increase in risk tolerance for lotteries with delayed payoffs. The consistently poor fit of Model (1) indicates that ignoring possible nonlinearities in value/utility functions may distort inferred discount rates; standard errors imply greater precision than is warranted. Also, consistent with previous research on intertemporal choice, implied discount rates were found to vary inversely with the length of the payoff delay.

6. Discussion and Suggestions for Future Research

The conventional approach to discounting implies complete intertemporal independence and reliance on market interest rates. What that implies in terms of intertemporal preferences is that no preference learning takes place. For example, our current and previous avenues of entertainment will have no bearing on our preferences for entertainment in the future. If these assumptions are valid, discount rates should not

differ over time or across outcome sign within a given decision option. Gain/loss asymmetry, on the other hand, predicts that negative and positive outcomes are likely to be treated differently over time. If gains are discounted faster, the riskiest choices we make may be the ones for which the consequences must be faced immediately. But if losses are discounted faster, then, indeed, "the riskiness of racetrack wagers [will decline] as post time approaches" (Jones & Johnson, 1973, p. 613).

The discount rates inferred in this study support the claim that, on average, the perceived severity of loss declines faster with delay than the perceived benefit of gain. This means that many decision makers who are risk averse in the short run will appear more risk tolerant for delayed outcomes. Previous research has shown that risk preferences are variable because they depend on context and decision frame, as well as on innate personality factors (e.g., Tversky & Kahneman, 1981; Lopes, 1987; March & Shapira, 1987). "[M]anagers", for example, "are inclined to show greater propensity toward risk taking when questions are framed as business decisions than when they are framed as personal decisions" (March & Shapira, 1987, p. 1409), and managers who define success or failure in terms of a particular target level of return may be relatively more (less) risk tolerant if their current position is below (above) the target (Lopes, 1987; March & Shapira, 1987). It now appears that payoff timing is a dimension of context that influences risk preferences.

March and Shapira (1987) note that managers do not define risk in terms of variance, nor attractiveness in terms of expected value; instead, many focus on the magnitude of potential loss to determine risk. Time discounting alone implies that the perceived magnitude of outcomes diminishes with temporal distance. The rates inferred in this study indicate that the perceived magnitude of loss diminishes faster than that of gain. This asymmetry implies that the tradeoff between the potential gain and loss components of a decision option will not be the same for distant as for immediate payoff times because gain will receive relatively more weight when payoffs are distant.

Similarly, estimated implicit risk rates imply that the perceived ability, willingness, or necessity, of paying future losses is treated as less certain than the ability to collect a future gain, at least for the present study. The subjective loss risk rates in Benzion et al. (1989) were also higher, so in both the present and the Benzion et al. studies, subjective losses were perceived as less certain (see Shelley, 1990, for an explanation of subjective loss in this setting). The question that emerges is: Why would decision makers discount future loss magn itudes more than gain magnitudes?

March & Shapira (1987) suggest an explanation indirectly. They note that there are "three pervasive features of managerial treatment of risk that deviate from simple conceptions of risk and are important for understanding managerial decision making" (March & Shapira, 1987, p. 1411). First, managerial risk conceptions are insensitive to probability estimates. "[I]t appears to be the magnitude of the value of the outcome that defines risk for managers, rather than some weighting of that magnitude by its likelihood" (March & Shapira, 1987, p. 1411).[2] Second, risk propensity tends to vary with the focus of the decision maker's attention, and attention varies with context (Mischel & Ebbesen, 1970; Mischel, Grusec, & Masters, 1969; Lopes, 1987; March & Shapira, 1987). For example, "[a]s a result of changing fortunes or aspirations, focus is shifted away from the dangers involved in a particular alternative and toward its opportunities" (March & Shapira, 1987, p. 1412; Lopes, 1987). Changes in fortunes or aspirations influence subjective values, but change takes time and aspirations are embedded in the future. Delay provides an opportunity to pursue aspirations and to control outcomes. Third, managers believe they can control the odds (i.e., change the odds; March & Shapira, 1987, p. 1414). It follows that managers may believe they can also control the magnitude of the downside risk, given time. In fact, in many cases judicious choices and well-placed effort will mitigate potential negative outcomes. As a consequence,

---

[2]In fact, the best-fitting SEV model for 16 of the 30 subjects was one that did not scale outcome magnitudes with probabilities. Eleven of 30 subjects gave much more weight to loss than to gain outcomes.

managers may discount the nagnitude of loss more because they believe (1) its likelihood is les certain, and (2) its magnitude can be changed over time.

Although the results obtained in this study indicate a tendency to discount losses and gains differently, on average, generalizations of this tendency to particular decision makers or decision situations should be made with caution. Risk propensity is known to vary with context. Time preferences may as well. In fact, both Fisher (1930) and Böhm-Bawerk (1923) predict they will. The experimental stimuli used in this study were relatively abstract. Richer contextual detail may exacerbate or mitigate the effect uncovered. For example, outcomes were all monetary in the present study; negative outcomes may be discounted quite differently when they are measured in terms of deaths, disease frequencies, or environmental decay, or when they are relatively fleeting as are electric shocks or routine business trips. Future research might be directed toward determining the effect of specific decision contexts or toward identifying the extent to which control of the odds and control of outcome magnitudes influence perceptions of risk for decisions with delayed consequences.

Appendix

Subject (Deleted Observation)	% of Explainable Variation Explained	df Lack of Fit/ Pure Error	F to test Lack of Fit	Prob > F
1 (17)	96.92	109/128 (108/127)	1.4056 (1.2465)	0.0320 (0.1160)
2	97.69	109/128	1.1316	0.2498
3	95.69	109/128	0.9397	0.6299
4	93.43	108/128	0.9150	0.6822
5 (26)	97.65	108/128 (107/127)	1.3430 (1.9974)*	0.0546 (0.0001)
6	98.08	109/128	0.8795	0.7545
7 (101)	98.35	108/128 (107/127)	1.4242 (0.9237)	0.0275 (0.6631)
8	96.74	108/128	0.9609	0.5832
9 (73)	94.50	108/128 (109/127)	1.2700 (1.1488)	0.0970 (0.2250)
10 (100)	97.83	108/128 (107/127)	2.7860 (1.0022)	0.0000 (0.4932)
11	97.19	109/128	0.9683	0.5673
12	94.73	109/128	1.8003*	0.0007
13	96.22	108/128	0.9271	0.6565
14	88.69	108/128	1.0050	0.4872
15	92.61	108/128	0.5101	0.9998

*Significant at $\alpha = 0.10$.

Table 1A
Tests for Lack of Model Fit: Full Approximation Model
Subjects 1 through 15

Subject (Deleted Observation)	% of Explainable Variation Explained	df Lack of Fit/ Pure Error	F to test Lack of Fit	Prob > F
16	94.96	108/128	0.7365	0.9491
17	96.38	109/128	1.1170	0.2727
18	93.46	109/128	0.7615	0.9282
19 (112)	93.85	108/128 (107/127)	1.2840 (1.1324)	0.0872 (0.2336)
20	95.76	108/128	0.9514	0.6040
21 (98)	92.08	108/128 (107/127)	1.2900 (1.1434)	0.0833 (0.2494)
22	97.79	108/128	0.7255	0.9569
23	93.94	108/128	1.0590	0.3764
24	96.55	109/128	0.7626	0.9271
25	91.28	108/128	0.7427	0.9443
26	93.33	108/128	1.2517	0.1112
27 (47)	98.39	108/128 (107/127)	1.3240 (1.2378)	0.0637 (0.1241)
28	96.11	108/128	1.1350	0.2451
29	91.57	109/128	1.5800*	0.0065
30	92.16	108/128	0.8017	0.8817

*Significant at α = 0.10.

Table 2A
Tests for Lack of Model Fit: Full Approximation Model
Subjects 16 through 30

References

Anderson, N. H. (1982). Foundations of information integration theory. New York: Academic Press.

______________. (1982). Methods of information integration theory. New York: Academic Press.

Benzion, U., Rapoport A., & Yagil, J. (1989). Discount rates inferred from decisions: An experimental study. Management Science, 35(3), pp. 270-284.

Birnbaum, M. H. (1978). Differences and ratios in psychological measurement. In J. Castellan & F. Restle (Eds.), Cognitive theory, Vol. 3). Hillsdale, NJ: Erlbaum.

Böhm-Bawerk, E. V. (1923). The positive theory of capital. (W. Smart, Trans.). New York: G. E. Stechert. (Original work published in 1888).

Buseymeyer, J. R. (1980). Importance of measurement theory, error theory, and experimental design for testing the significance of interactions. Psychological Bulletin, 88, pp. 237-244.

Conard, J. W. (1963). An introduction to the theory of interest. Berkeley: University of California Press.

deBoor, C. (1978). A practical guide to splines. New York: Springer-Verlag.

Debreu, G. (1959). Theory of value. New Haven: Yale University Press.

Draper, N. R., & Smith, H. (1981). Applied regression analysis. (2nd ed.). New York: Wiley.

Fisher, I. (1930). The theory of interest. New York: Macmillan.

Gallant, A. R. (1987). Nonlinear Statistical Models. New York: Wiley & Sons.

Henderson, J. M., & Quandt, R. E. (1980). Microeconomic theory: A mathematical approach. (3rd ed.). New York: McGraw-Hill.

Jones, E. E., & Johnson, C. A. (1973). Delay of consequences and the riskiness of decisions. Journal of Personality, 41, pp. 613-637.

Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. Econometrica, 47(2), pp. 263-291.

Loewenstein, G. F. (1988). Frames of mind in intertemporal choice. Management Science, 34(2), pp. 200-214.

______________, and Prelec, D. (1989a) Decision making over time and under uncertainty: a common approach. Unpublished working paper. Center for Decision Research, Graduate School of Business, University of Chicago.

______________, and Prelec, D. (1989b) Anomalies in intertemporal choice: evidence and an interpretation. Unpublished working paper. Center for Decision Research, Graduate School of Business, University of Chicago.

______________, and Thaler, R. (1989) Anomalies, intertemporal choice. Journal of economic perspectives, 3(4).

Lopes, L. (1987). Between hope and fear: The psychology of risk. Advances in experimental social psychology, 20, pp. 255-295.

March, J., and Shapira, Z. (1987). Managerial perspectives on risk and risk taking. Management Science, 33(11), pp. 1404-1418.

Meyer, R. F. (1975). State-dependent time preference. In Proceedings of the workshop on decision making with multiple conflicting objectives (pp. 232-244). International Institute for Applied Systems Analysis, Laxenburg, Austria.

Miller, N. E. (1959) Liberalization of basic stimulus-response concepts: extension of conflict behavior, motivation, and social learning. In S. Koch (Ed.), Psychology: A study of science, 2. New York: McGraw-Hill.

Mischel, W., & Ebbesen, E. B. (1970). Attention in delay of gratification. Journal of Personality and Social Psychology, 16, pp. 329-337.

______________, Grusec, J., & Masters, J. C. (1969). Effects of expected delay time on the subjective value of rewards and punishments. Journal of Personality and Social Psychology, 11, pp. 363-373.

Nisan, M. (1972). Dimension of time in relation to choice behavior and achievement orientation. Journal of Personality and Social Psychology, 21, pp. 175-182.

______________, & Minkowich, A. (1973). The effect of expected temporal distance on risk taking. Journal of Personality and Social Psychology, 25, pp. 375-380.

Schumaker, L. (1981). Spline functions: Basic theory. New York: Wiley

Shelley, M. K. (1990) Outcome signs, question frames, and discount rates. Unpublished manuscript. University of Illinois at Urbana-Champaign.

Stevenson, M. K. (1986). A discounting model for decisions with delayed positive or negative outcomes. Journal of Experimental Psychology: General, 115(2), pp. 131-154.

Strotz, R. H. (1955). Myopia and inconsistency in dynamic utility maximization. Review of Economic Studies, 23, pp. 165-180.

Thaler, R. (1981). Some empirical evidence on dynamic inconsistency. Economics Letters, 8, pp. 201-207.

Tversky, A., & Kahneman, D. (1981). The framing of decisions and the psychology of choice. Science, 211, pp. 453-458.

Winsberg, S., & Ramsay, J. O. (1981). Analysis of pairwise preference data using integrated B-splines. Psychometrica, 46, pp. 171-186.